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Fast construction of good rank-1 lattice rules for multivariate numerical integration in weighted function spaces

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In this talk, we consider multivariate numerical integration in weighted reproducing kernel Hilbert spaces by means of quasi-Monte Carlo cubature methods. In particular, we focus on so-called rank-1 lattice rules for integration in weighted spaces of s-variate periodic functions with prescribed decay of the Fourier coefficients and associated smoothness α. Rank-1 lattice rules are fully specified by their generating vector $\boldsymbol{z} \in \mathbb{Z}^s$ and the number of points N and it is known that there exists lattice rules which achieve the (almost) optimal error convergence order of $\mathcal{O}(N^{-\alpha})$ in the respective function spaces. For the construction of generating vectors of such “good” lattice rules, one often resorts to (greedy) computer search algorithms such as the component-by-component (CBC) construction for which fast implementations are available, see, e.g., [3]. Here, we investigate as a generalization of the CBC algorithm the so-called (reduced) successive coordinate search (SCS) construction, see [1] and [2], with the aim of constructing even better generating vectors. For the two considered algorithms we present worst-case error bounds that prove the (almost) optimal error behavior and derive suitable conditions on the involved weight parameters to make the implied constants independent of the dimension. Furthermore, we show that the algorithms can be implemented in a fast manner requiring $\mathcal{O}(sN \ln N)$ or less operations. We illustrate these theoretical findings by numerical experiments.

[1] A. Ebert, H. Leövey, D. Nuyens. *Successive coordinate search and component-by-component construction of rank-1 lattice rules.* In P. Glynn and A. B. Owen, editors, Monte Carlo and Quasi-Monte Carlo Methods 2016, 197–215, Springer, Berlin, 2018.

[2] A. Ebert, P. Kritzer. *Constructing lattice points for numerical integration by a reduced fast successive coordinate search algorithm.* Journal of Computational and Applied Mathematics, 351:77–100, 2019.

[3] D. Nuyens, R. Cools. *Fast algorithms for component-by-component construction of rank-1 lattice rules in shift-invariant reproducing kernel Hilbert spaces.* Mathematics of Computation, 75(254):903–920, 2006.