Approximating Borel measures by Dirac measures

For every integer $N \geq 1$ and all dimensions $d \geq 1$ there exist finite sets $x_1, \ldots, x_N \subset [0,1]^d$ whose star-discrepancy with respect to the Lebesgue measure is of order at most $(\log N)^{d-1} N^{-1}$. Recently, Aistleitner, Bilyk, and Nikolov showed that for any normalized Borel measure μ, there exist finite sets whose star-discrepancy with respect to μ is at most $(\log N)^{d-\frac{1}{2}} N^{-1}$. In the Borel case, even for discrete measures, very little else is known. In this talk, we provide for dimension $d = 1$ an explicit construction of sets that achieve discrepancy no worse than the Lebesgue measure, confirming the conjecture that when $d = 1$ the Lebesgue measure is the hardest measure to approximate. In dimension $d \geq 2$ the situation is much more complicated. Nonetheless, we can also in this case identify a large family of (finite or infinitely supported) discrete measures that is easier to approximate than the Lebesgue measure indicating that the Lebesgue is indeed also the hardest one to approximate if $d \geq 2$. This is joint work with Samantha Fairchild and Max Goering.