Optimal recovery of functions from subsampled random points

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I will present the results of our joint work with David Krieg, Mario Ullrich and Tino Ullrich [1] on the recovery of functions based on function evaluations.

We provide a method to transfer results for L_2-approximation to rather general seminorms (including L_p, $1 \leq p \leq \infty$). The underlying (least squares) algorithm is based on a random construction and subsampling based on the solution of the Kadison-Singer problem.

Besides an explicit bound for the corresponding sampling widths, we also obtain some interesting inequalities between the sampling, Kolmogorov and Gelfand widths. Namely, we show that for a compact topological space D and a compact subset F of $C(D)$ the following holds

$$g_{2n}^{\text{lin}}(F, L_\infty) \ \leq \ 115 \sqrt{n}\, d_n(F, L_\infty). \tag{1}$$

The bound in (1) is optimal up to constants, also if we consider only convex and symmetric F and replace the Kolmogorov width d_n by the Gelfand width c_n on the right hand side.

This means, that there are linear algorithms using $2n$ samples that are as good as all algorithms using arbitrary linear information up to a factor of at most $\sqrt{n}$. A result that cannot be true without oversampling [2]. Moreover, our results imply that linear sampling algorithms are optimal up to a constant factor for many reproducing kernel Hilbert spaces.

1. Krieg D., Pozharska K., Ullrich M., Ullrich T. Sampling recovery in the uniform norm, arXiv: 2305.07539v2, 2023.
2. Novak E. Deterministic and stochastic error bounds in numerical analysis, Lecture Notes in Mathematics 1349, Springer-Verlag, 1988.