A Toolbox for Fast Harmonic Approximation on the Rotation Group

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Abstract

Band-limited functions on the rotation group $\mathcal{SO}(3)$ can be described by the Fourier coefficients of its harmonic series. Therefore, operations like addition or convolution are directly computable on frequency domain.

For the evaluation of such harmonic series there exist $\mathcal{O}(N^3 \log^2 N)$ algorithms [2] [3]. Moreover, following Risbo [4] we get a polynomial transformation directly from a representation property of Wigner-D functions. This implies a practically faster $\mathcal{O}(N^4)$ algorithm. A combination of Gauß and Clenshaw-Curtis quadrature yields an inverse algorithm to compute the Fourier coefficients from the function evaluations on a (w.r.t Euler angles) equispaced grid. Using inner symmetries of functions on $\mathcal{SO}(3)$ we can improve these algorithms. The tangent space of a function on $\mathcal{SO}(3)$ is isomorphic to $\mathbb{R}^3$. By a clever choice of the basis of the tangent space, the gradient of a harmonic series can be described by 3 harmonic series. This only requires scaling and shifted addition of the Fourier coefficients.

References

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