English Version of: **Aufgaben zur Förderung metakognitiver Kompetenzen,** Beiträge zum Mathematikunterricht 2010, S. 481–484. Münster: WTM

Christa KAUNE, Elmar COHORS-FRESENBORG, Osnabrück
Silke KRAMER, Springe

Tasks to encourage metacognitive competencies

1. Introduction

In the international mathematics education discussion increasing metacognitive activities in the classroom is considered to be an important measure to improve the sustainability of the mathematics education: It is the approach to teaching that is to be changed in such a way that the learners increasingly often plan, monitor and reflect their own thinking processes.

A concept taking account of the above is going to be developed and tested in a two-year collaborative project between the Institute for Cognitive Mathematics (University of Osnabrück) and the Otto-Hahn-Gymnasium (Springe). It is based on successful ideas from the project „*Mathematik Gut Unterrichten*" (*Teaching Mathematics well* Kaune et al., 2010) funded by the Deutsche Telekom Foundation. In particular more exercises are going to be designed aiming to increase the learners' metacognitive competencies. This is to support the development of a metacognitive and discursive approach to teaching. The effectiveness of this complex implementation will be investigated by a control group design.

2. Example tasks to evoke planning activities

While tasks that stimulate the learners to engage in monitoring and reflection can be found in recent German textbooks (see e.g. Lergenmüller and Schmidt, 2001) tasks that should trigger the students' planning activities are very seldom found. One can find however suggestions encouraging the learner to follow strategies (see Griesel et al., 2006, p. 109). This is in line with international efforts to train students to use those strategies (Mevarech et al., 1997). But it seems to present some difficulties for the learners to adopt those strategies as their own ones without each time being told to by the teacher (Depaepe et al., 2010). In order to change this one must influence the teaching culture in general. As a starting point appropriate tasks could serve that motivate the students to engage in autonomous planning activities. A teacher can accomplish this for example in a situation in which a student has been missing (because of illness) by assigning the following homework to the whole class:

Please write down for Dennis what we have been working on in class today, so that he is able to do his homework without any further assistance.

"As you haven't been in class today, you probably can't solve the exercise without a strategy. Since I am friendly I'm going to tell you my strategy. Accordingly I recommend you to first remove the parentheses. Then I would combine the terms. I would do this

A student chooses the example: 3 (x + 4) - 10 = 7x + 10 +3 x and highlights at the beginning of her help the need for a strategy. The next sentences describe plans for activities, such as: remove *whenever possible. After this I suggest you to rearrange the terms, that is on the one side the x-terms, on the other side the normal numbers. When you have done this, I would keep on calculating in such a way that on the left hand side you are left with only · x [times x] and on the right hand side only with a normal one [number]. Then I divide by the number that is multiplied by x.”*

parentheses, combine like terms, rearrange the terms. Then she plans intermediate results: on one side the x-terms, on the other side the x-terms, on the other side the normal numbers, a term "times x ", just a normal number. At the end, she plans again an activity: to divide by the number. that is multiplied by x.

The self-directed formulation of the strategy requires that the student has reflected on her experience in solving equations. The way she precisely addresses her classmate suggests that she shows this behaviour also in the classroom-discourse.

The following second example task shows how mathematical expertise, thereon based cognitive processes, and the metacognitive activities controlling them are interwoven in demanding tasks. It is a further development of exercise 11 taken from Lergenmüller and Schmidt (2001, p. 205).

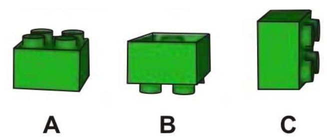
If a Lego brick with four knobs falls to the ground, it can show “4 eyes”, “1 eye” or it might land sideways. In the table you can find different predictions for the probability of the outcomes A, B and C.

a) In your opinion, which of the estimates are good ones, which aren't? Please justify your opinion.

b) Plan how one could determine the probabilities?

c) Write down your own estimate.

d) Carry out your plan. Whose estimate is the best one?



	A	B	C
Inge	0,33	0,33	0,33
Peter	0,4	0,4	0,2
Jutta	0,4	0,55	0,05
Hans	0,4	0,6	0
Fred	0,3	0,5	0,1

Analysing the parts of the exercise and student solutions we refer to Cohors-Fresenborg and Kaune (2007).

<u>Group 1</u>:

“We think that Hans is right, because the probability that the brick lands on the side (C) is very small. It is possible to put the brick on its side face, but being thrown it never stays standing on the side. But we suspect that possibility A occurs most often, because the knobs make it heavier and pull it down. Inge on the other hand, we don't believe she is right, because it never occurs that the brick stays standing as often as a & b).”

Working on part a) one has to control both subject-specific activities and the plausibility of the result as well as one has to check the chosen modelling approach. Reflective judgements are also necessary in order to justify the answer, as the solutions of two groups of students show.

Group 2:

"We don't agree with any of the estimates, because it is more probable, that a Lego brick lands like A, as then the largest face touches the ground. This is also the reason why B is more probable than C. Inge and Fred don't even come up to a 100 with all percentages."

By the means of b) the learners are first encouraged to engage in planning activity. The teaching culture established in this learning group requires that they justify their activities. This demands reflecting activities, as the following elaborations show:

Group 3:

"Our plan:

We thought, since we are three people, that 2 throw the Lego brick 33 times and one throws the brick 34 times, because then we have thrown the brick 100 times and we also have got so to say 100% and then it is easier to calculate the percentages. (In class one could make three groups. Because then we have so to say thrown the dice a thousand times and got more outcomes.)"

This group of students plans (well-grounded) a method to prove a prognosis before they reflect on calculation advantages. A second group also plans a method and justifies it later by reflecting on its effectiveness.

Group 4:

" b) Everyone throws the Lego stone 30 times, then the results are gathered and the relative frequencies calculated. The relative frequency will be close to the probability, because the Lego brick has been thrown 900 times."

Parts c) and d) initially demand activities on the content level. Subsequently a reflective assessment is expected.

3. Conclusion

The analysis of the example tasks and the students' solutions revealed how metacognitive activities contribute to meet the arguing and modelling competences that are called for in the standards for education. To improve the quality of teaching it is not sufficient to provide the teaching staff with appropriate exercises. Students will only argue in written in a well-founded justified manner, if this is also part of the discourse in class. In the project *Mathematik Gut Unterrichten* it has been shown how the awareness of the interaction between the students' and teachers' cognitive and metacognitive activities could be raised in a teacher coaching by involving the teaching staff (Kaune et al. 2010).

Literature

Cohors-Fresenborg, E. and Kaune, C. (2007): *Kategoriensystem für metakognitive Aktivitäten beim schrittweise kontrollierten Argumentieren im Mathematikunterricht*. Arbeitsbericht Nr. 44. Osnabrück: Forschungsinstitut für Mathematikdidaktik.

Depaepe, F. et al. (2010): Teachers' metacognitive and heuristic approaches to word problem solving: analysis and impact on students beliefs and performance. *ZDM: The International Journal on Mathematics Education*, 42(2), 205–218.

Griesel, H. et al.. (2006): *Elemente der Mathematik 7 Niedersachsen*. Hannover: Schroedel.

Kaune, C. and Cohors-Fresenborg, E. (Ed.) (2010): *Mathematik Gut Unterrichten - Analyse von Mathematikunterricht bezüglich metakognitiver und diskursiver Aktivitäten*. Osnabrück: Forschungsinstitut für Mathematikdidaktik.

Lergenmüller, A. and Schmidt, G. (2001): *Mathematik neue Wege 7*. Braunschweig: Westermann.

Mevarech, Z. R. and Kramarski, B. (1997): IMPROVE: A Multidimensional Method for Teaching Mathematics in Heterogeneous Classrooms. *American Educational Research Journal,* 34(2), 365–394.